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Modeling of Failure and Response to Laminated Composites Subjected to In-Plane Loads

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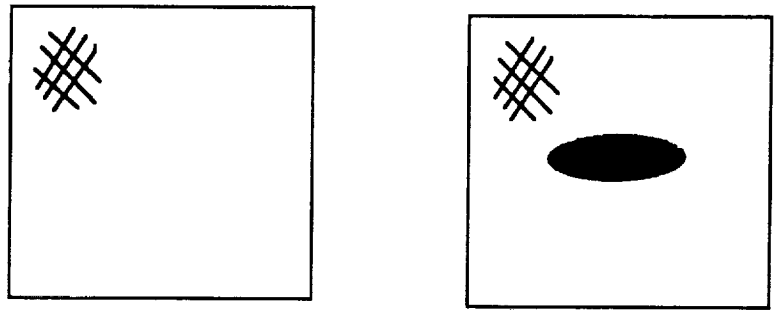
CURRENT STATUS

An analytical model has been developed for predicting the response of laminated composites with or without a cutout and subjected to in-plane tensile and shear loads. Material damage resulting from the loads in terms of matrix cracking, fiber-matrix shearing, and fiber breakage was considered in the model. Delamination, an out-of-plane failure mode, was excluded from the model.



TENSION SHEAR

GIVEN : GEOMETRY, LAYUP, LOADS

PREDICT:

- **DAMAGE IN COMPOSITES**
  - MATRIX CRACKING
  - FIBER-MATRIX SHEARING
  - FIBER BREAKAGE
- **RESIDUAL STIFFNESS**
- **RESPONSE AS A FUNCTION OF LOADS**
- **FAILURE**

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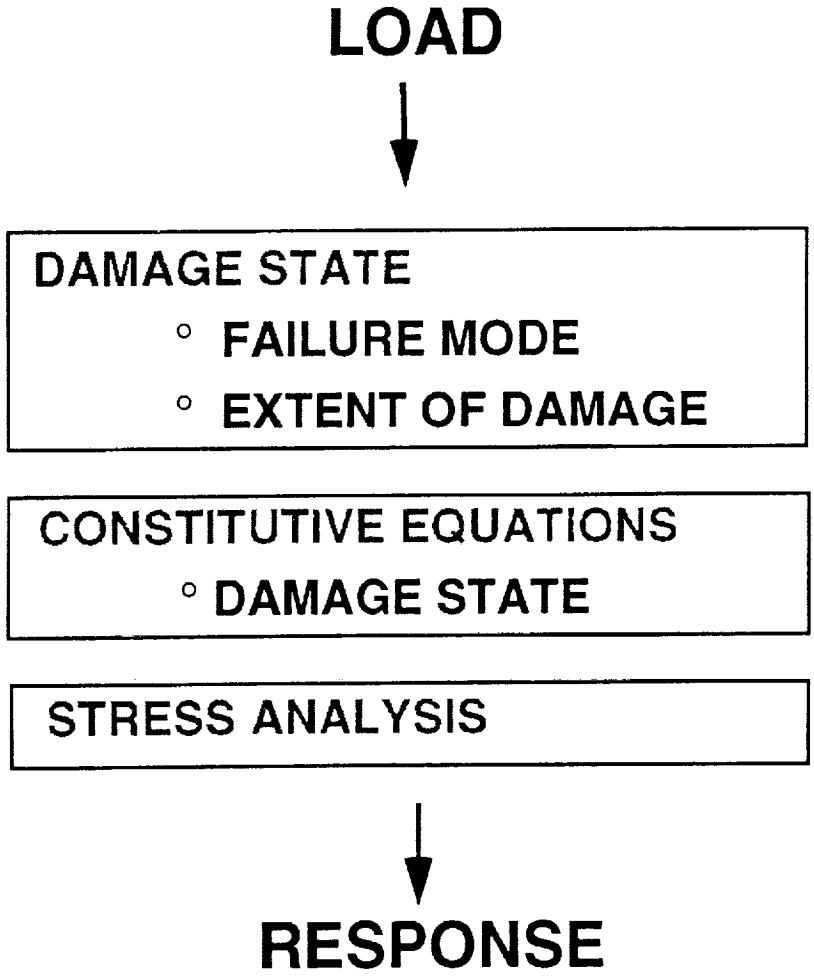
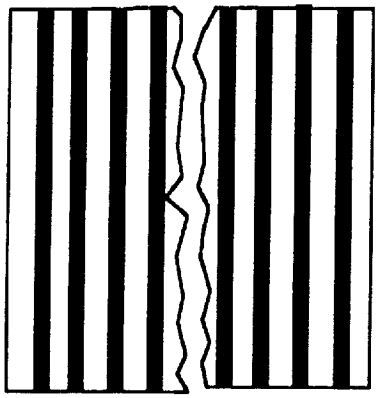
WHAT DO WE NEED?

In order to accurately predict the response of the laminates, the model must be capable of predicting the state of damage as a function of the applied load, relating the damage state to the loss of material properties, and calculating stresses and strains everywhere inside the materials. Accordingly, the proposed analytical model consists of three parts: constitutive modeling, failure analysis and stress analysis.

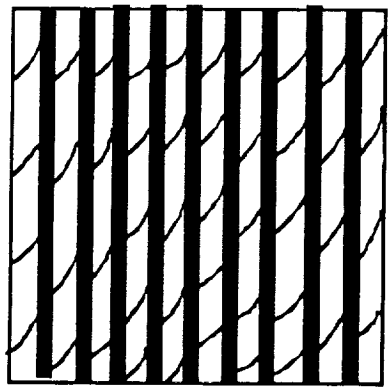


FAILURE MODES

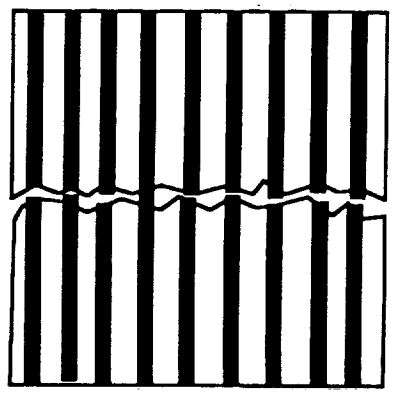
The three basic in-plane failure modes of a single unidirectional ply considered in the model are matrix cracking, fiber-matrix shearing, and fiber breakage.



MATRIX CRACKING



FIBER-MATRIX SHEAR-OUT



FIBER BREAKAGE

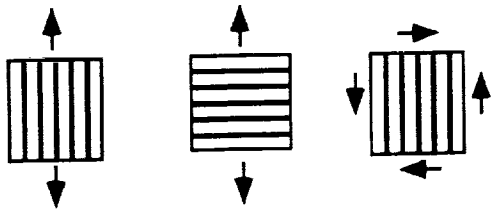
CONSTITUTIVE MODELING

The constitutive equations of a unidirectional ply in an undamaged state can be characterized by standard mechanical testing. However, once damage occurs in a ply within a multidirectional laminate, the material properties of the ply need to be determined in order to construct the constitutive equations for the damaged laminate. Therefore, the proposed model was based on continuum mechanics whereby the damaged ply in a laminate was treated as a continuous body with degraded material properties.

PLY STIFFNESS (UNDAMAGED STIFFNESS, DAMAGE STATE)

UNDAMAGED PLY:

MECHANICAL TESTING



$\rightarrow E_x, E_y, E_s, \nu_{xy} \rightarrow [Q]$

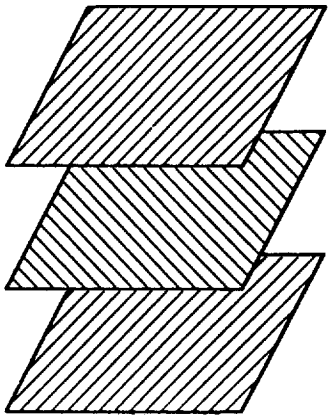
DAMAGED PLY: (IN LAMINATE)

- MATRIX CRACKING
- FIBER-MATRIX SHEAR-OUT
- FIBER BREAKAGE

$\overset{D}{E_x}, \overset{D}{E_y}, \overset{D}{E_s}, \overset{D}{\nu_{xy}} \rightarrow [Q^D] = ?$

MATRIX CRACKING

In order to determine the effect of matrix cracking on the reduction of the stiffness of a unidirectional ply in a laminate, crack density was selected as the damage parameter for characterizing the damage state of matrix cracking.



LAMINATION EFFECT

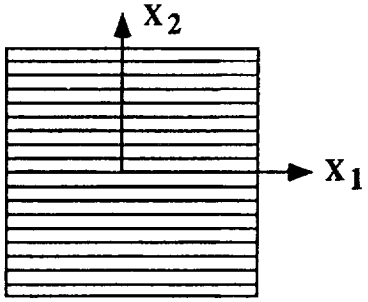
↓

MATRIX CRACK DENSITY

(ϕ)

↓

FOR EACH PLY



EFFECTIVE PLY STIFFNESS

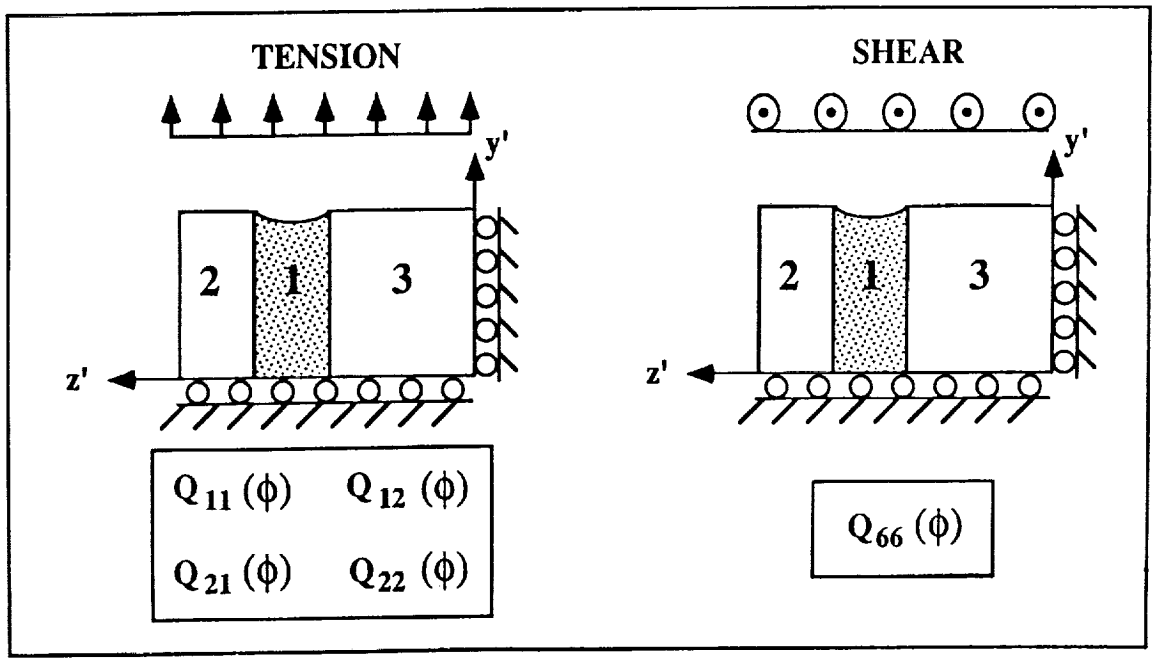
$$[Q^D] = \begin{bmatrix} Q_{11}(\phi) & Q_{12}(\phi) & 0 \\ Q_{21}(\phi) & Q_{22}(\phi) & 0 \\ 0 & 0 & Q_{66}(\phi) \end{bmatrix}$$

C-2

MATRIX CRACKING - APPROACH

A constitutive model was developed for characterizing the material properties of a ply in a symmetric laminate as a function of its own crack density. For a given crack density in a ply whose fiber direction may not be parallel to the global x-axis, the model first rotates the laminate such that the fiber direction of the cracked ply is aligned with the x-axis. It is then assumed that all the matrix cracks in the ply are uniformly distributed. As a result, a unit-cell of the laminate can be selected as a representative volume of the cracked laminate. The representative volume may be comprised of up to three sublaminates labeled as 1, 2 and 3 in the figure.

- ORTHOTROPIC SUBLAMINATES ASSUMPTION
- 2-D ELASTICITY ANALYSIS

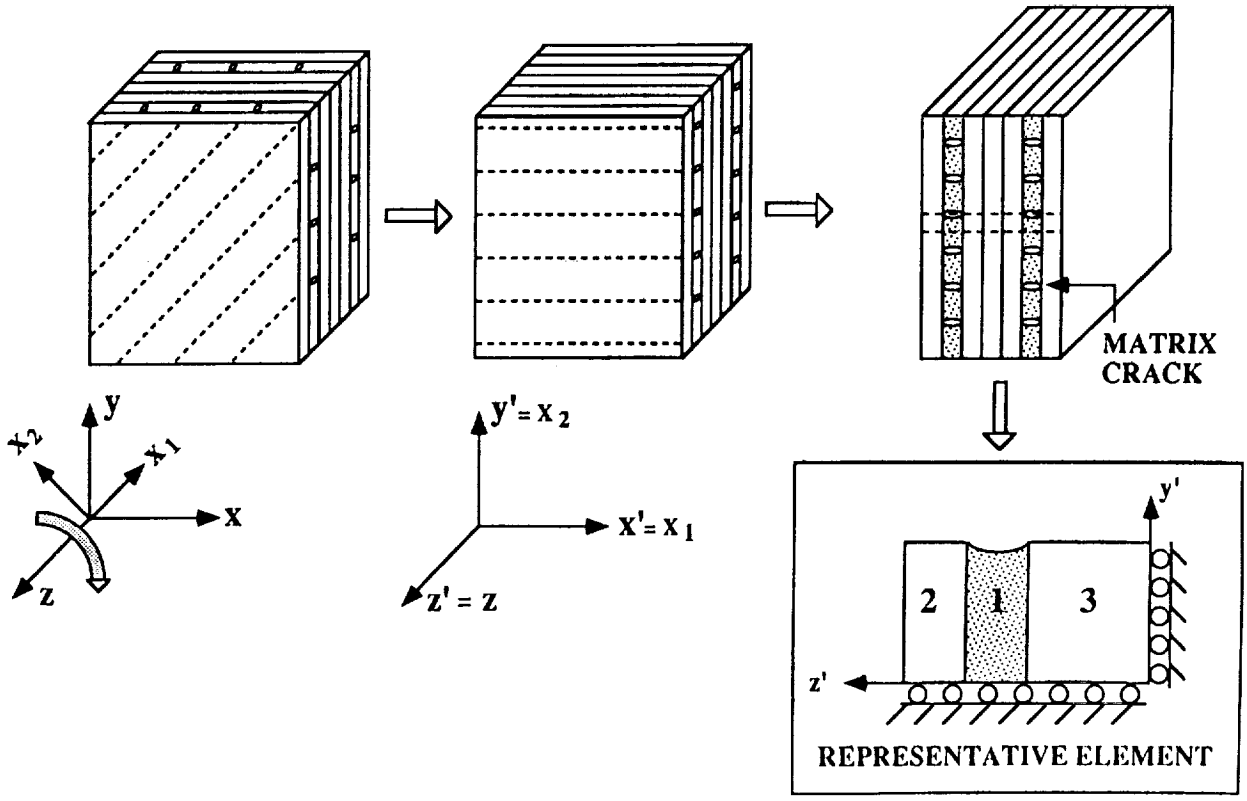


- REPEAT PROCEDURE FOR ALL PLIES OF THE LAMINATE

MATRIX CRACKING

In the constitutive model it was further assumed that the sublaminates 2 and 3 could be treated as homogeneous and orthotropic materials. Accordingly, the three-dimensional volume could be reduced to a two-dimensional element. By applying a far-field tensile or shear load, the material properties of the cracked ply (sublaminate 1) as a function of the crack density could be calculated from a two-dimensional elasticity theory. The aforementioned procedure was then applied to each of the plies in a laminate for any given crack density.

APPROACH



FIBER-MATRIX SHEAR-OUT

Once the applied load continued to increase, the plies in the laminate may have failed due to either fiber-matrix shearing or fiber breakage, leading to catastrophic failure of the laminate. Fiber-matrix shear-out failure could be attributed to interfacial debonding and slipping or nonlinear elasticity of the material. The aforementioned elasticity theory for matrix cracks could not be applied to characterize the reduction of material properties resulting from the shear-out failure. To account for interfacial debonding and slipping, continuum damage mechanics was adopted based on the concept proposed by Krajcinovic and Fonseka. Nonlinear material response was considered in the model through the shear stress-shear strain relationship.

HIGH SHEAR DEFORMATION

↓

- FIBER-MATRIX INTERFACE DEBONDING, SLIP ETC.
- NONLINEAR SHEAR DEFORMATION

- CONTINUUM DAMAGE MECHANICS: (Krajcinovic and Fonseka, 1981)

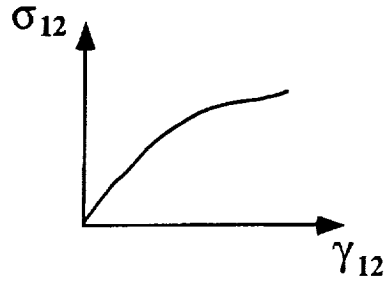
$$Q_{66}^{D} = Q_{66}(\phi)\, d_s \qquad d_s = e^{-\left(\frac{\phi}{\phi_0}\right)^{\eta}}$$

ϕ_0 = SATURATION CRACK DENSITY

η = SHAPE PARAMETER

- PLY SHEAR STRESS-SHEAR STRAIN:

$$\underline{\text{Hahn:}} \quad \gamma_{12} = \frac{\sigma_{12}}{Q_{66}^{D}} + \alpha \left(\frac{\sigma_{12}}{Q_{66}^{D}}\right)^{3}$$



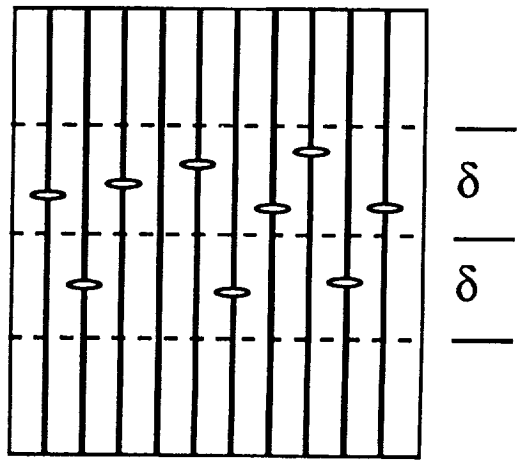
FIBER BREAKAGE

Based on Rosen's cumulative weakening failure theory, failure of a unidirectional ply under tension occurs only when there are enough fiber breaks that occur within a critical area characterized by the fiber interaction distance δ, which is the maximum distance within which one fiber break would affect the stresses of the neighboring fibers. Accordingly, not only stresses but also the area within which fiber breaks occur are essential for characterizing fiber failure of a unidirectional composite.

UNIDIRECTIONAL COMPOSITE:

CUMULATIVE WEAKENING FAILURE (Rosen, 1964)

FAILURE OF UNIDIRECTIONAL PLY OCCURS AT THE WEAKEST CROSS SECTION

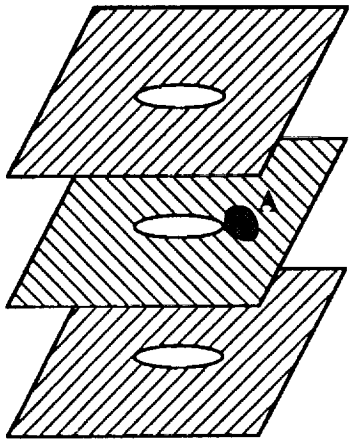


δ ← FIBER INTERACTION ZONE

FIBER BREAKAGE

A hypothesis was postulated that stiffness reduction of a unidirectional composite due to fiber breakage is related to the extent of the area in which fiber breakage occurs.

NOTCHED COMPOSITE:



HYPOTHESIS:

STIFFNESS REDUCTION IS FUNCTION OF FIBER BREAKAGE AREA (A)

$$d_f = e^{-\left(\frac{A}{\delta^2}\right)^{\beta}}$$

A = FIBER BREAKAGE AREA

β = SHAPE PARAMETER

$$[Q^D] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} d_f & 0 & 0 \\ 0 & d_f & 0 \\ 0 & 0 & d_f \end{bmatrix}$$

CONSTITUTIVE MODEL

The effective material properties of a single ply within a symmetric laminate can be related to undamaged material properties and damage state with three different failure modes.

- **WITHOUT SHEAR NON-LINEARITY**

$$\{\sigma\} = [Q^D]\{\varepsilon\}$$

$$[Q^D] = \begin{bmatrix} Q_{11}(\phi) & Q_{12}(\phi) & 0 \\ Q_{21}(\phi) & Q_{22}(\phi) & 0 \\ 0 & 0 & Q_{66}(\phi) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d_s \end{bmatrix} \begin{bmatrix} d_f & 0 & 0 \\ 0 & d_f & 0 \\ 0 & 0 & d_f \end{bmatrix}$$

MATRIX CRACKING — FIBER-MATRIX SHEAR-OUT — FIBER BREAKAGE

- **WITH SHEAR NON-LINEARITY**

$$\{d\sigma\} = [Q^D]^t\{d\varepsilon\}$$

DAMAGE GROWTH CRITERIA

Modified Hashin Failure Criteria were adopted for predicting the mode and state of damage of a ply in a laminate. The stresses used in the criteria are the effective stresses obtained from the effective properties. The effective strengths of the ply are no longer treated as constants, but may vary as a function of crack density (damage state).

PREDICT MODE OF FAILURE AND DAMAGE STATE

MATRIX CRACKING

$$\left(\frac{\sigma_{22}}{Y_t(\phi)}\right)^2 + \left(\frac{\sigma_{12}}{S(\phi)}\right)^2 \geqq 1$$

FIBER-MATRIX SHEAR-OUT

$$\left(\frac{\sigma_{11}}{X_t}\right)^2 + \left(\frac{\sigma_{12}}{S(\phi)}\right)^2 \geqq 1$$

FIBER BREAKAGE

$$\left(\frac{\sigma_{11}}{X_t}\right)^2 \geqq 1$$

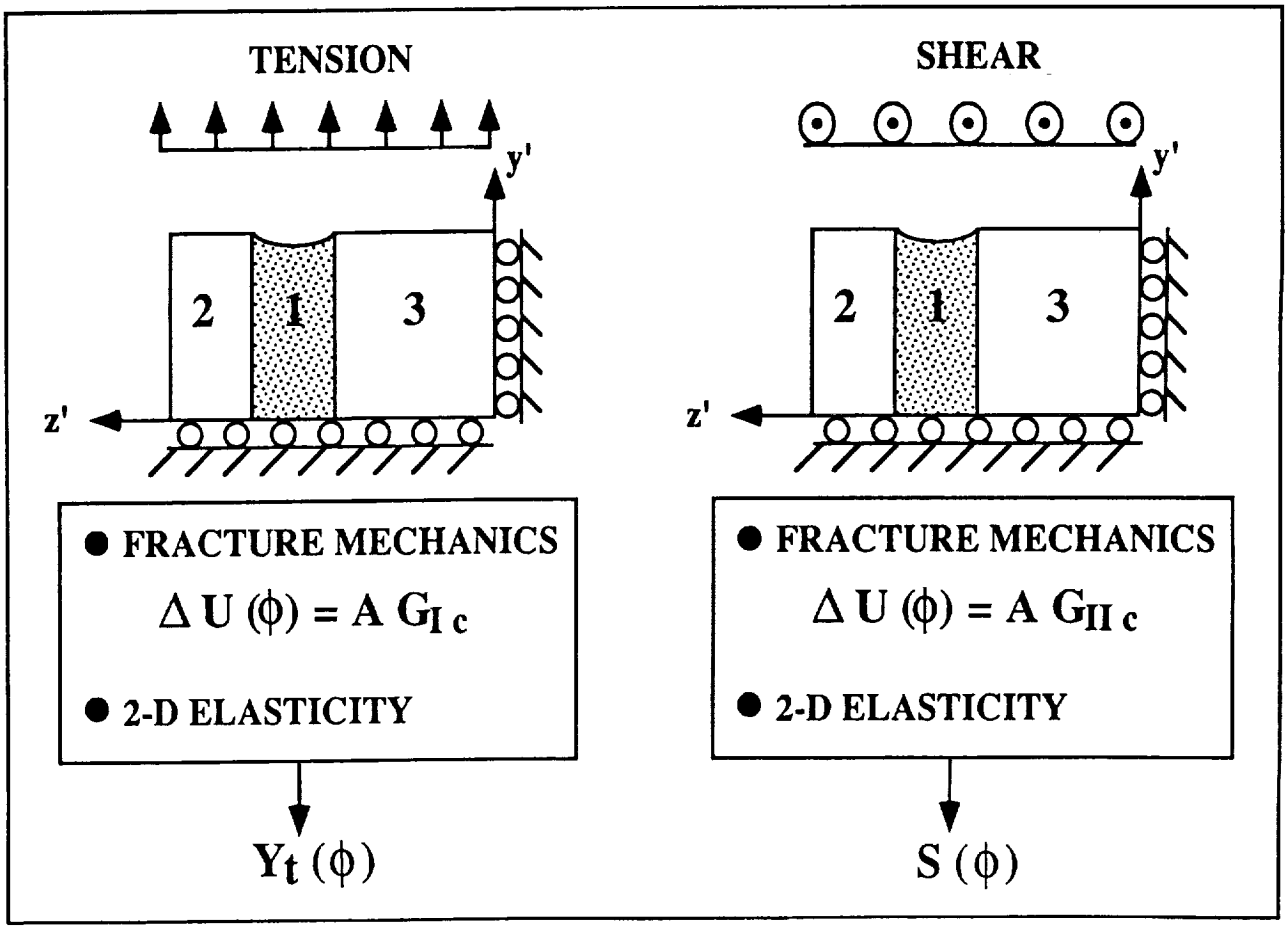
$Y_t(\phi) = ?$ $\quad$ $S(\phi) = ?$

EFFECTIVE STRENGTHS

The effective transverse tensile and shear strengths at crack density ϕ are defined as the minimum stresses that are required to generate crack density ϕ in the ply. A model was proposed based on the elasticity theory and fracture mechanics to characterize the effective strengths as a function of crack density.

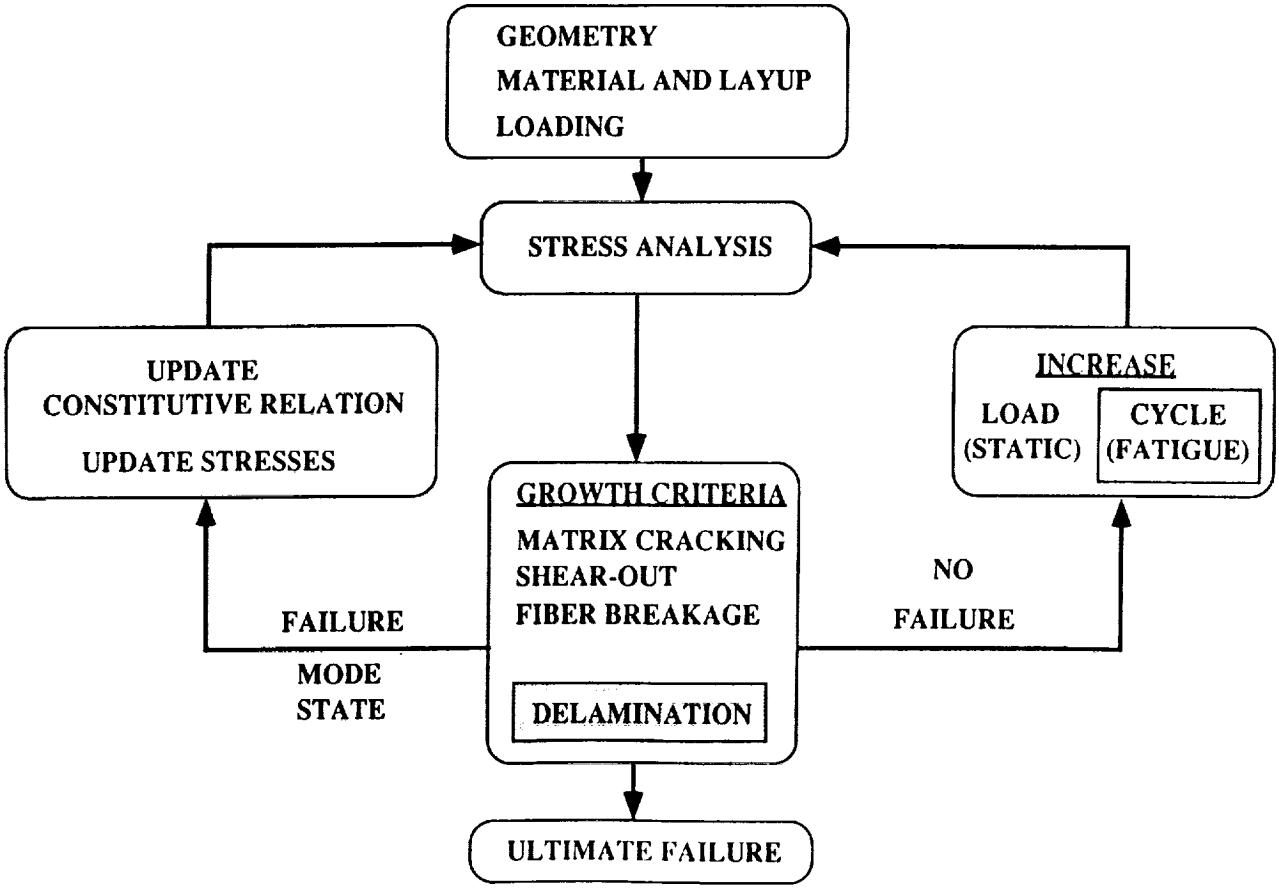
$Y_t(\phi)$ = MINIMUM TRANSVERSE STRESS REQUIRED TO GENERATE CRACK DENSITY ϕ

$S(\phi)$ = MINIMUM SHEAR STRESS REQUIRED TO GENERATE CRACK DENSITY ϕ



FLOWCHART

A finite element analysis has been developed based on the proposed model. The flowchart of the analysis is presented.



AS4/3501 $[0/90_2]_s$

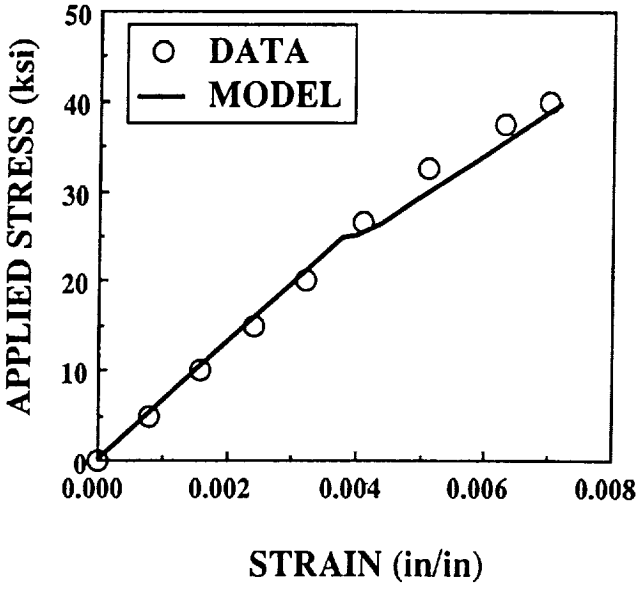
Comparison between the model prediction and the test data. A $[0/90_2]_s$ composite subjected to a 10^o off axis uniaxial tensile load.

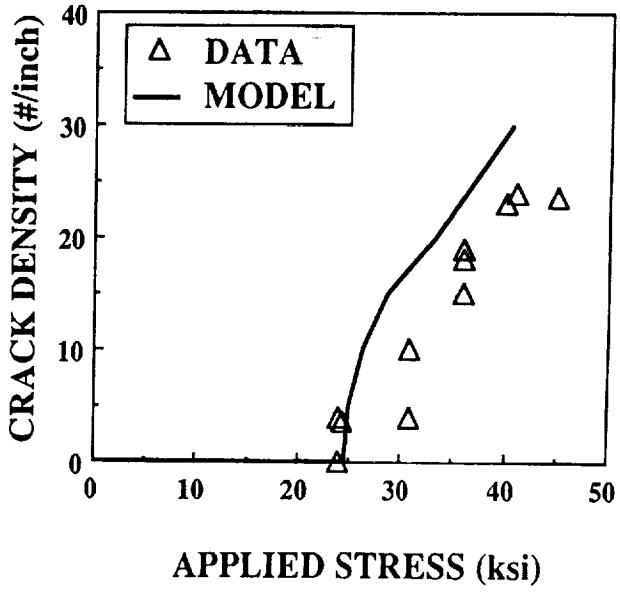
AS4/3501-6

$[0/90_2]_S$

10° Off-Axis Tensile Loading

(Daniel and tsai, 1991)





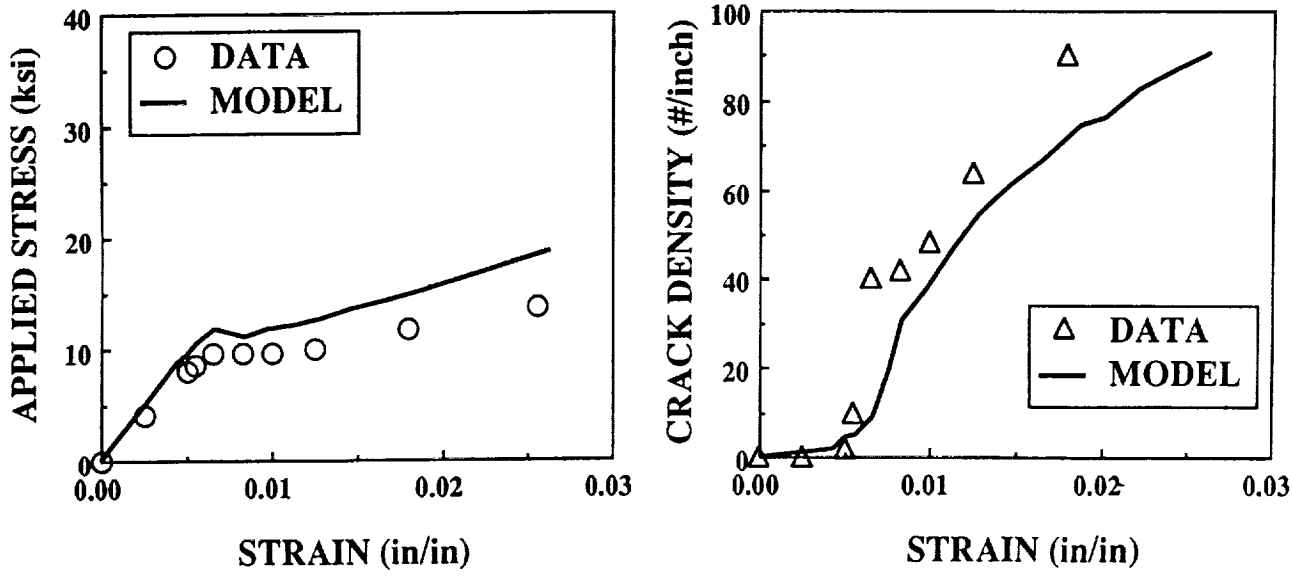
AS4/3502 $[60/90/-60/60/90/-60/90]_s$

Comparison between the model prediction and the test data. A $[60/90/-60/60/90/-60/90]_s$ composite subjected to a uniaxial tensile load.

AS4/3502

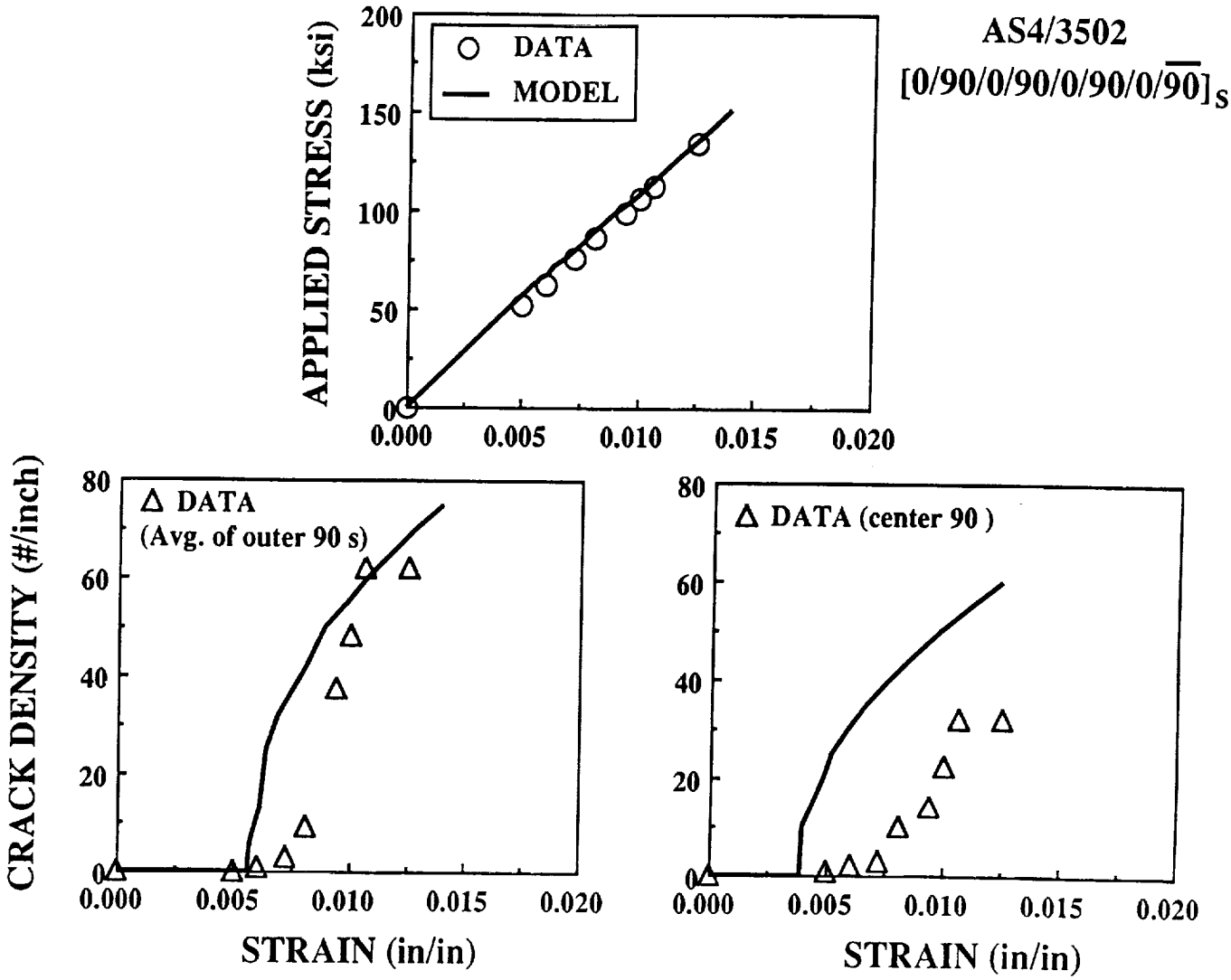
$[60/90/-60/90/60/90/-60/\overline{90}]_S$

(Kistner et al., 1985)

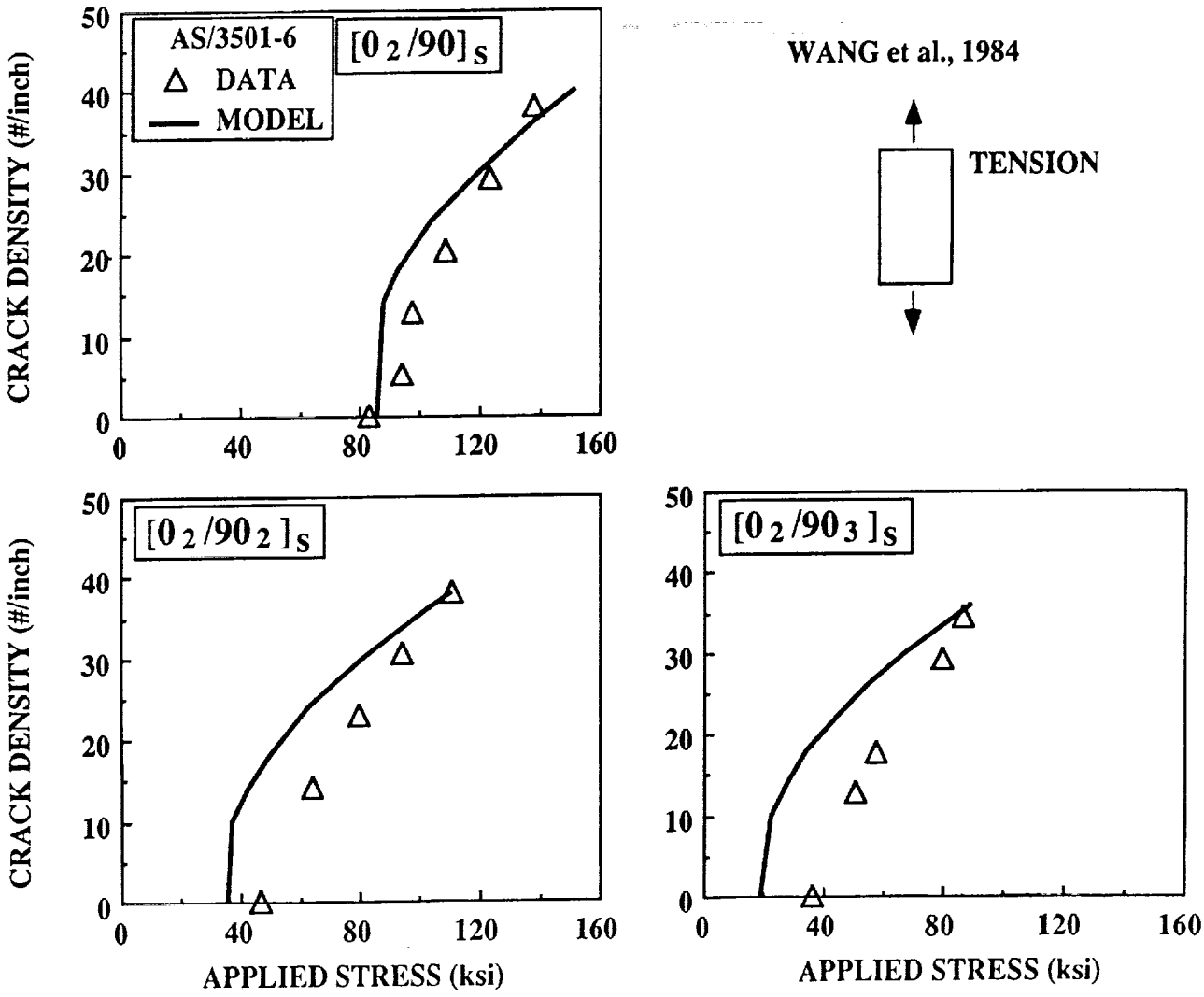


AS4/3502 $[0/90/0/90/0/90/0/90]_s$

Comparison between the model prediction and the test data. A $[0/90/0/90/0/90/0/90]_s$ composite subjected to a uniaxial tensile load.



Comparison between the model prediction and the test data. Cross-ply composites subjected to a uniaxial tensile load.



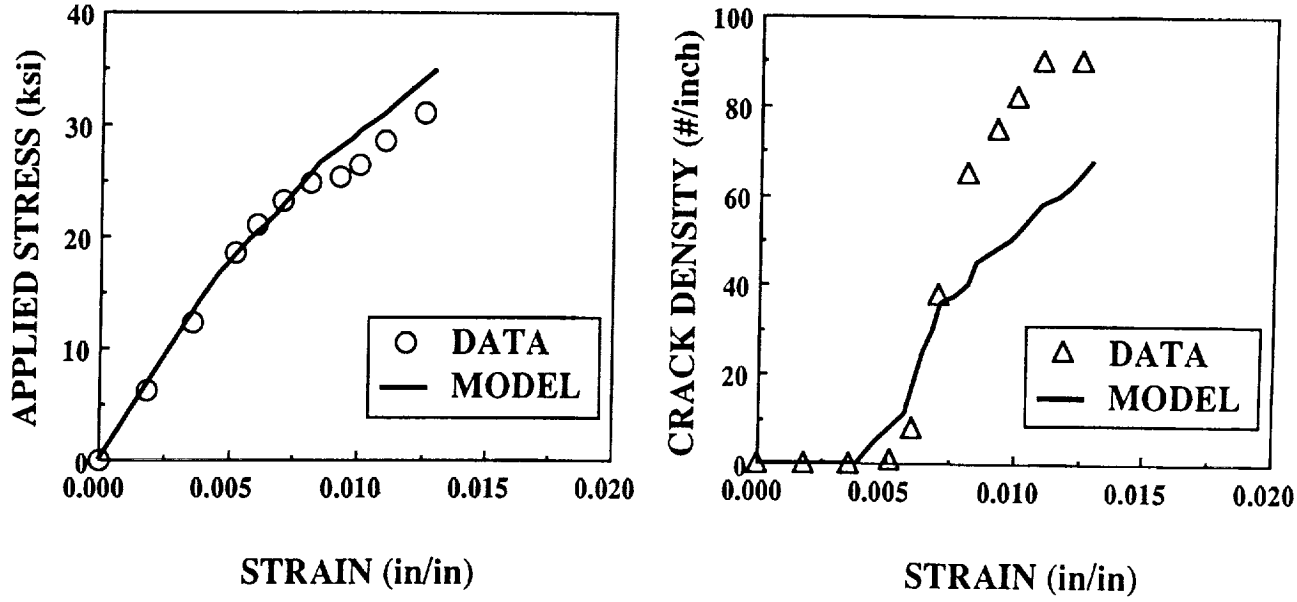
AS4/3502 $[45/90/-45/90/45/90/-45/90]_s$

Comparison between the model prediction and the test data. A $[45/90/-45/90/45/90/-45/90]_s$ composite subjected to a uniaxial tensile load.

AS4/3502

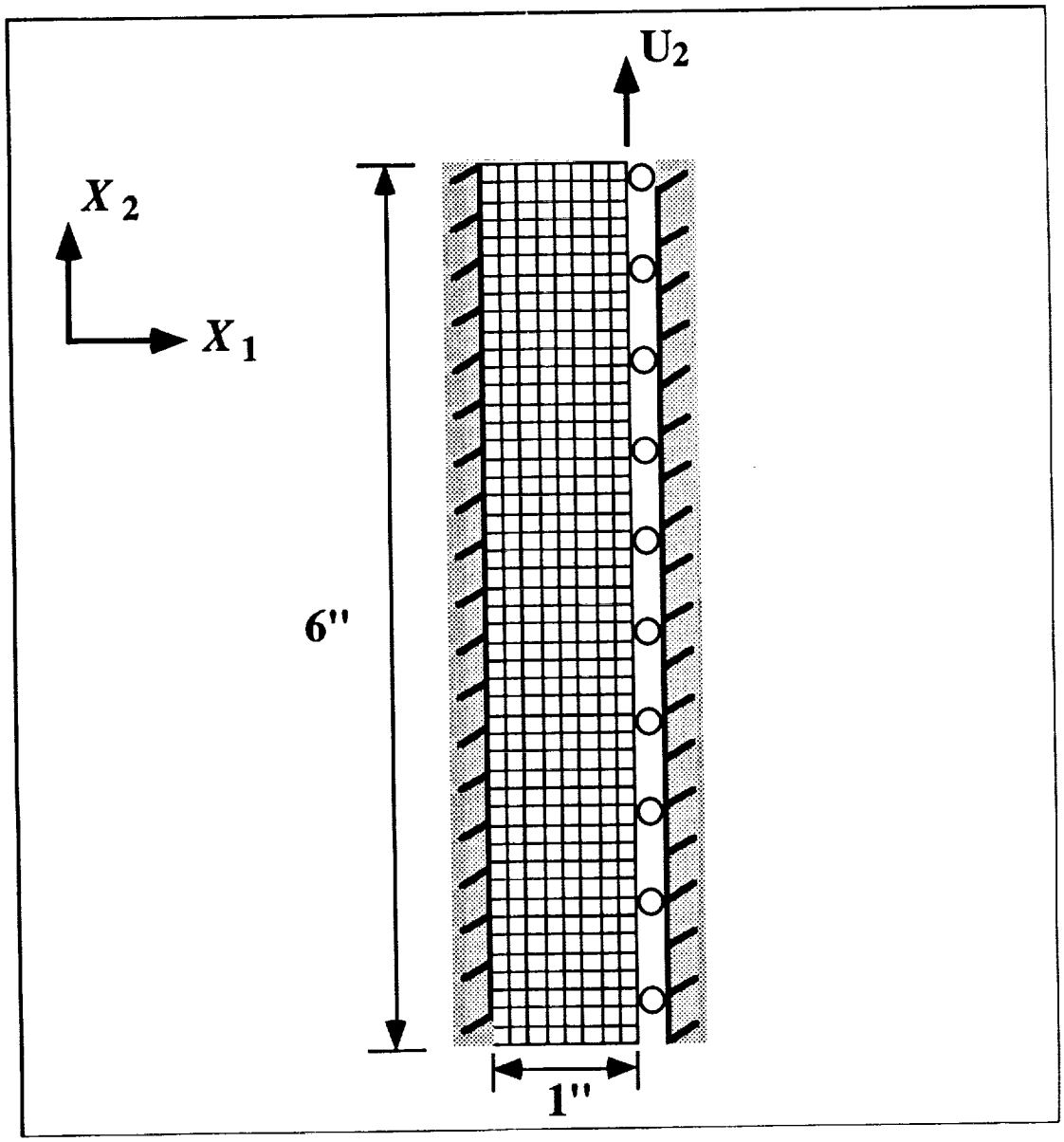
$[45/90/-45/90/45/90/-45/\overline{90}]_S$

(Kistner et al., 1985)



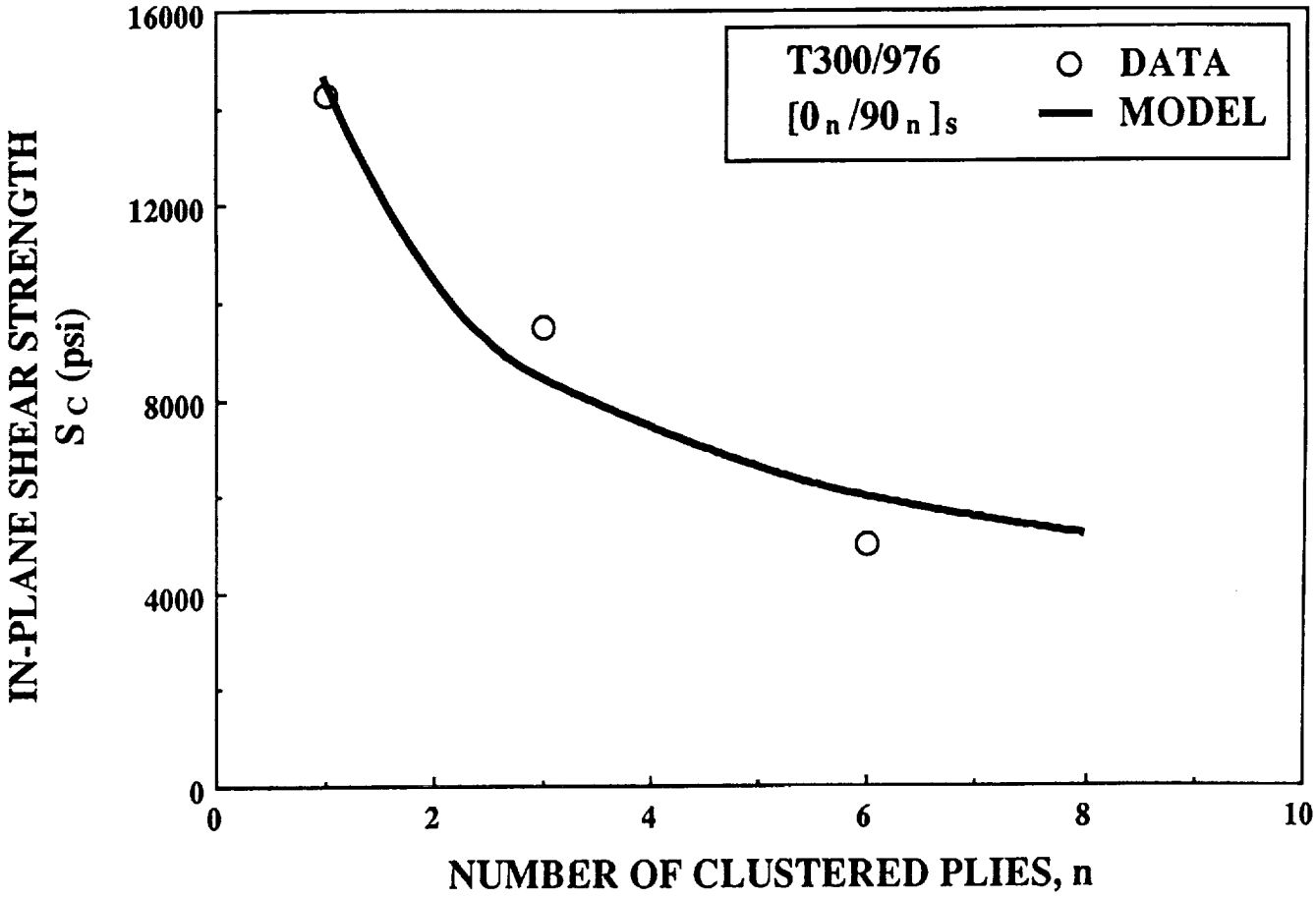
RAIL SHEAR SPECIMEN

A typical finite element mesh used in the calculation for rail shear specimens.



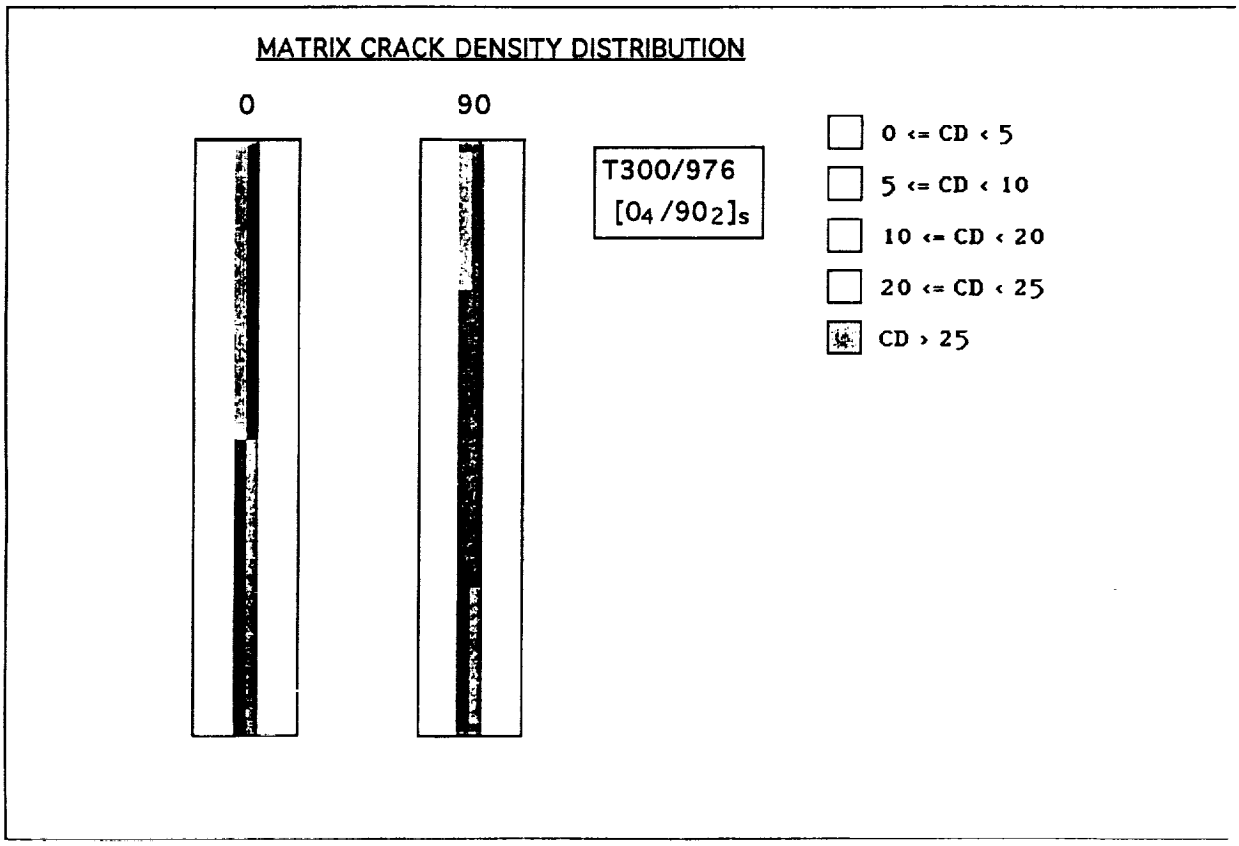
IN-PLANE SHEAR STRENGTH

Comparison between the prediction of rail shear strength and the measurement.



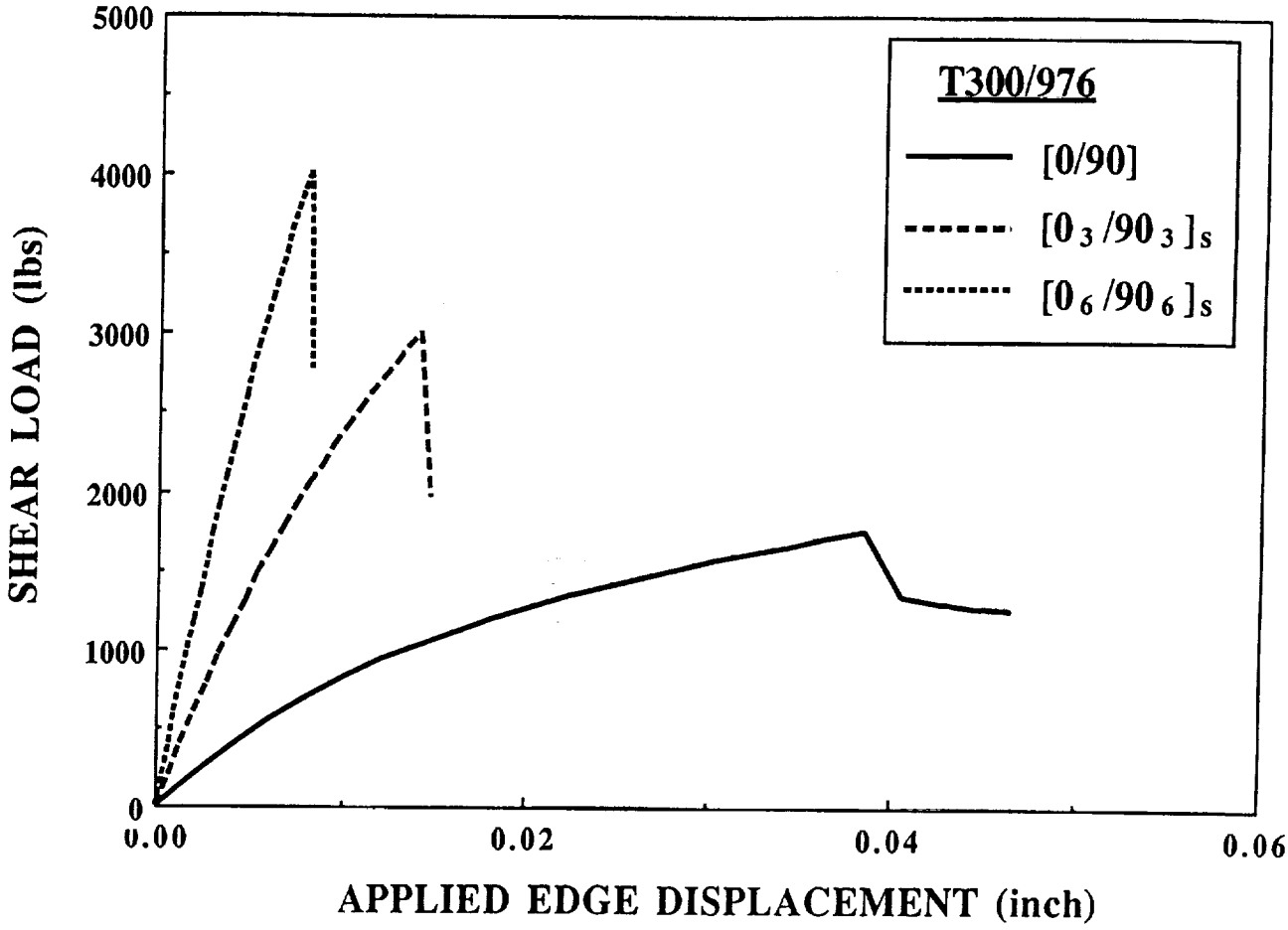
RAIL SHEAR TEST SIMULATION

The predicted matrix crack density distribution in a $[0_4/90_2]_s$ shear specimen near 90% of the final failure load.



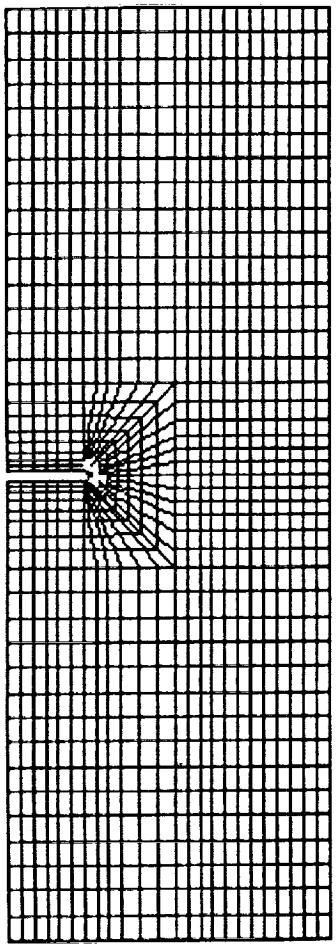
SHEAR LOAD

The predicted load-deflection response of cross-ply rail shear specimens.



PROGRESSIVE FAILURE PREDICTION (VERIFICATION - NOTCHED LAMINATE)

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



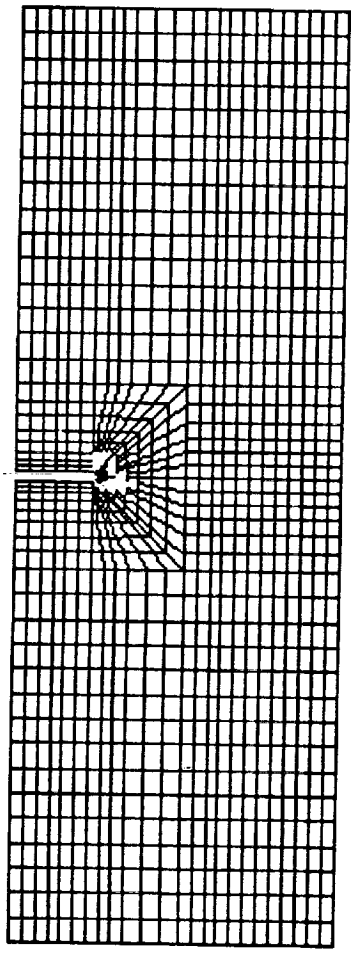
MATERIAL: AS4/3501-6
LAYUP: [45/90/-45/0]s
D= 0.872(in)
W/D= 4.0

LOAD = 5328(lbs)

DAMAGE MODE
Fiber Breakage
Fiber-Matrix Shear-Out
Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



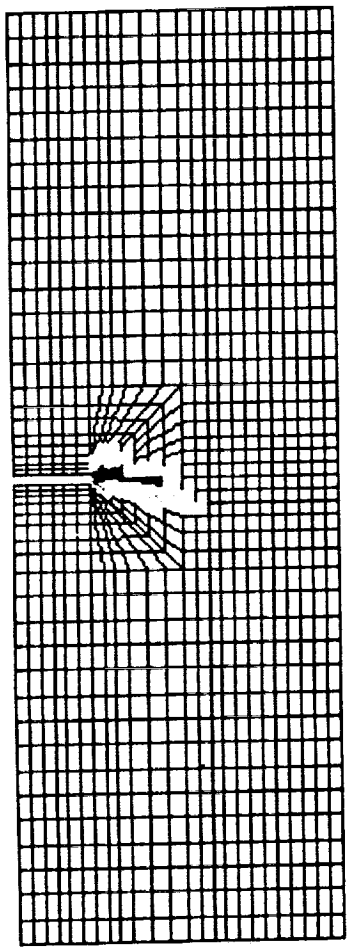
MATERIAL: AS4/3501-6
LAYUP: [45/90/-45/0]s
D= 0.872(in)
W/D= 4.0

LOAD = 6720(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



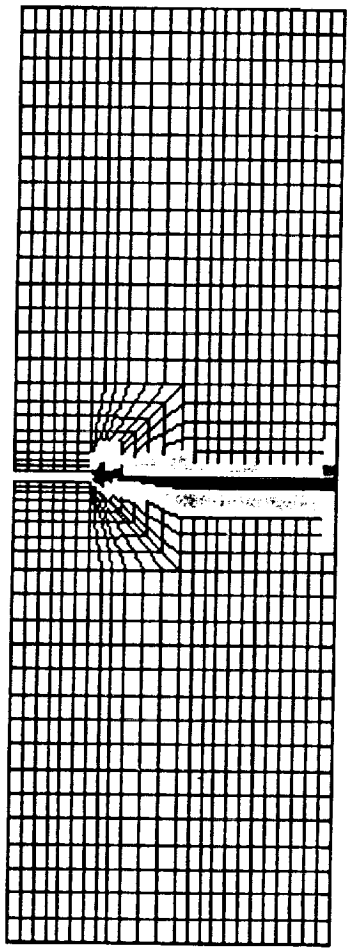
MATERIAL: AS4/3501-6
LAYUP: [45/90/-45/0]s
D= 0.872(in)
W/D= 4.0

LOAD = 7447(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



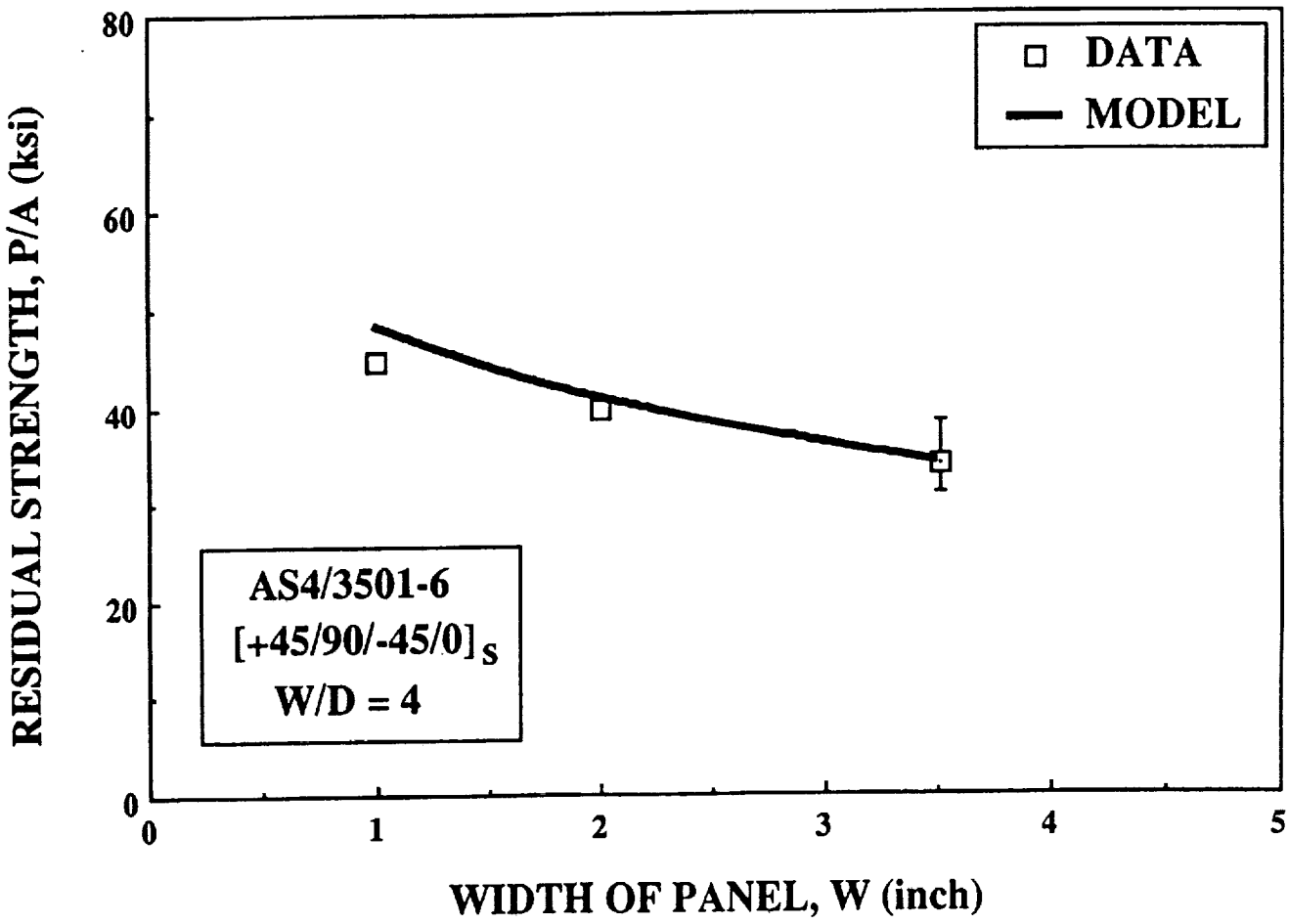
MATERIAL: AS4/3501-6
LAYUP: [45/90/-45/0]s
D= 0.872(in)
W/D= 4.0

LOAD = 3847(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

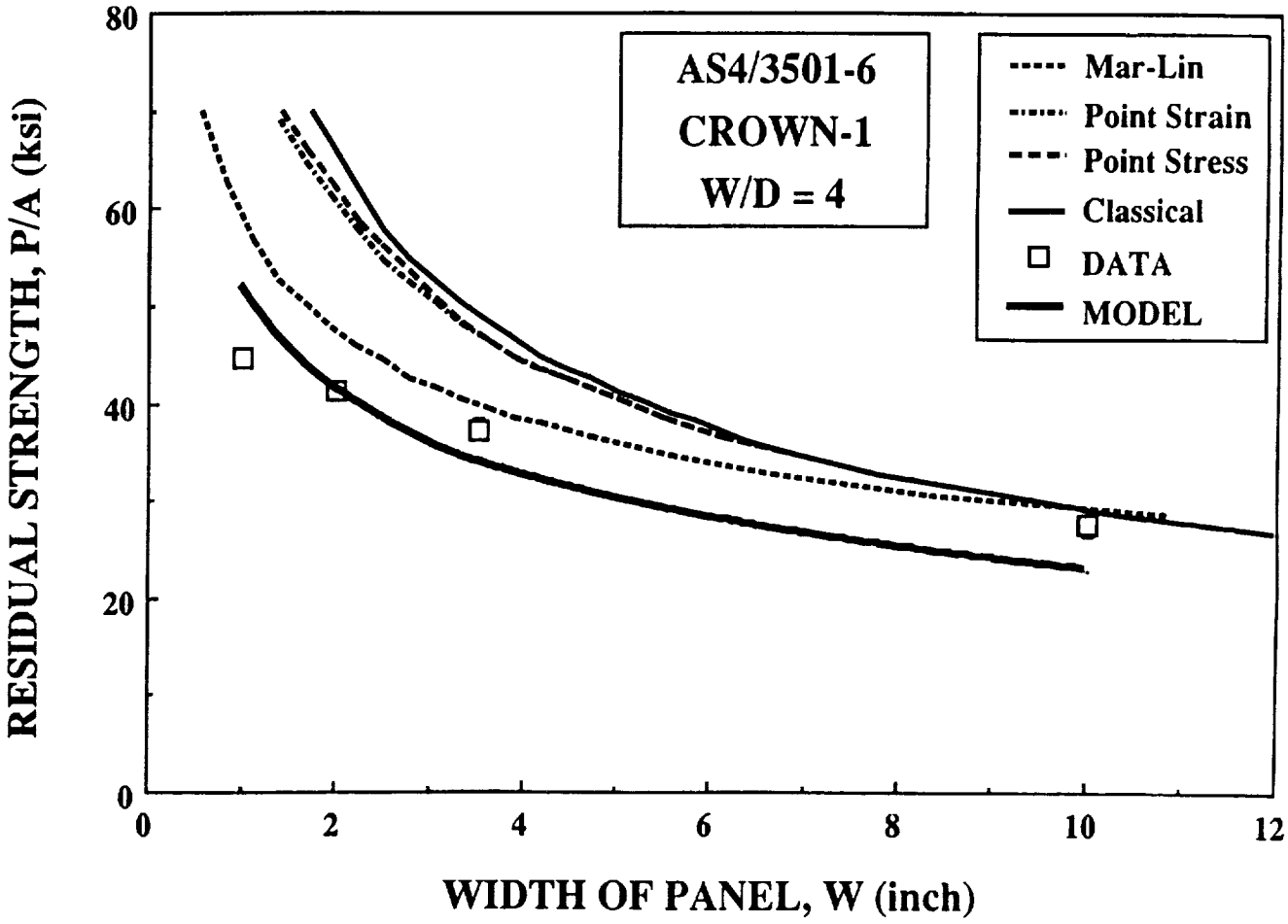
RESIDUAL STRENGTH

The residual strength distribution of notched $[45/90/-45/0]_s$ composites as a function of laminate width. Comparison between the prediction and the test data.



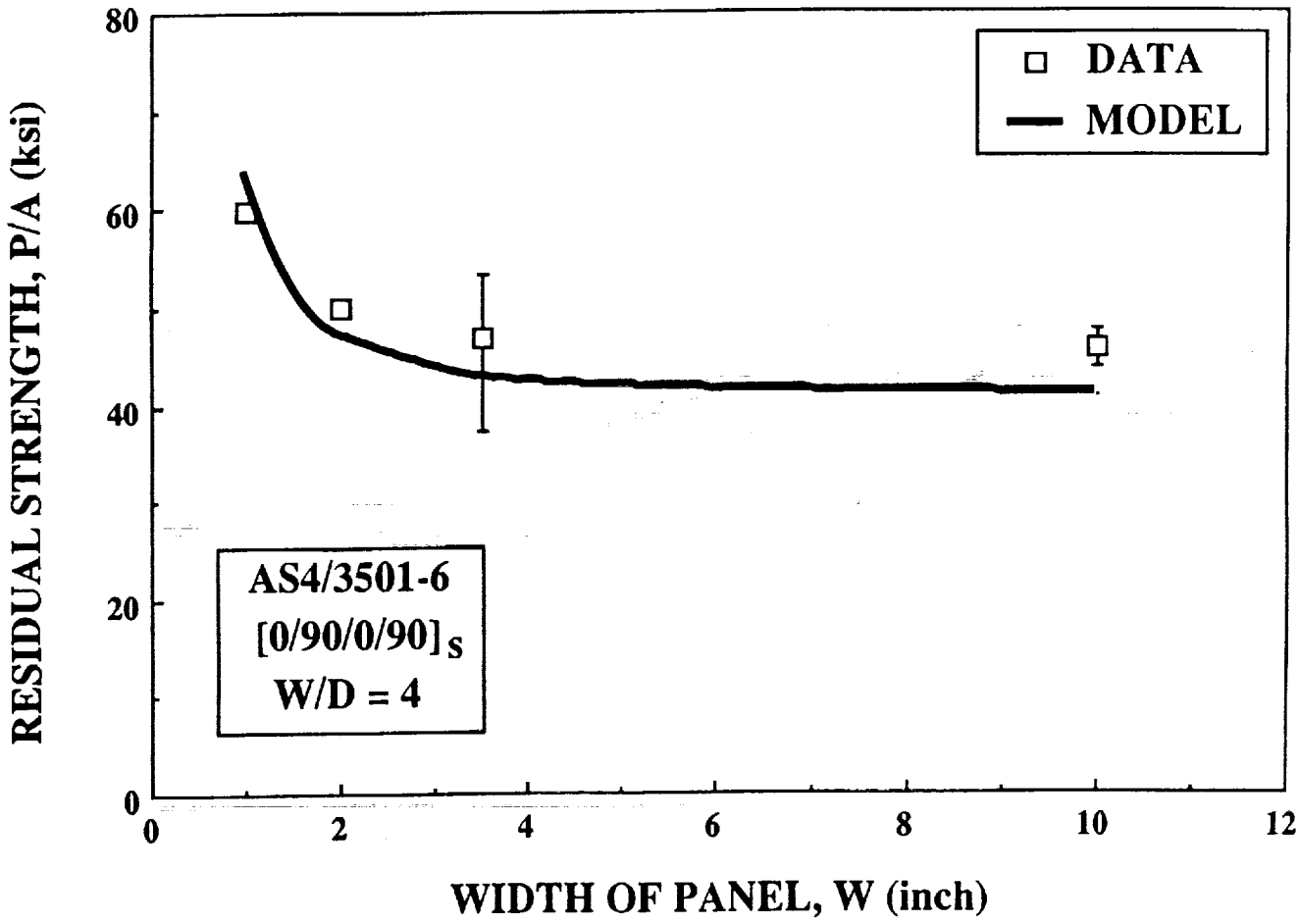
RESIDUAL STRENGTH

The residual strength distribution of notched [Crown-1] composites as a function of laminate width. Comparison between the predictions based on the model and the existing methods and the test data.



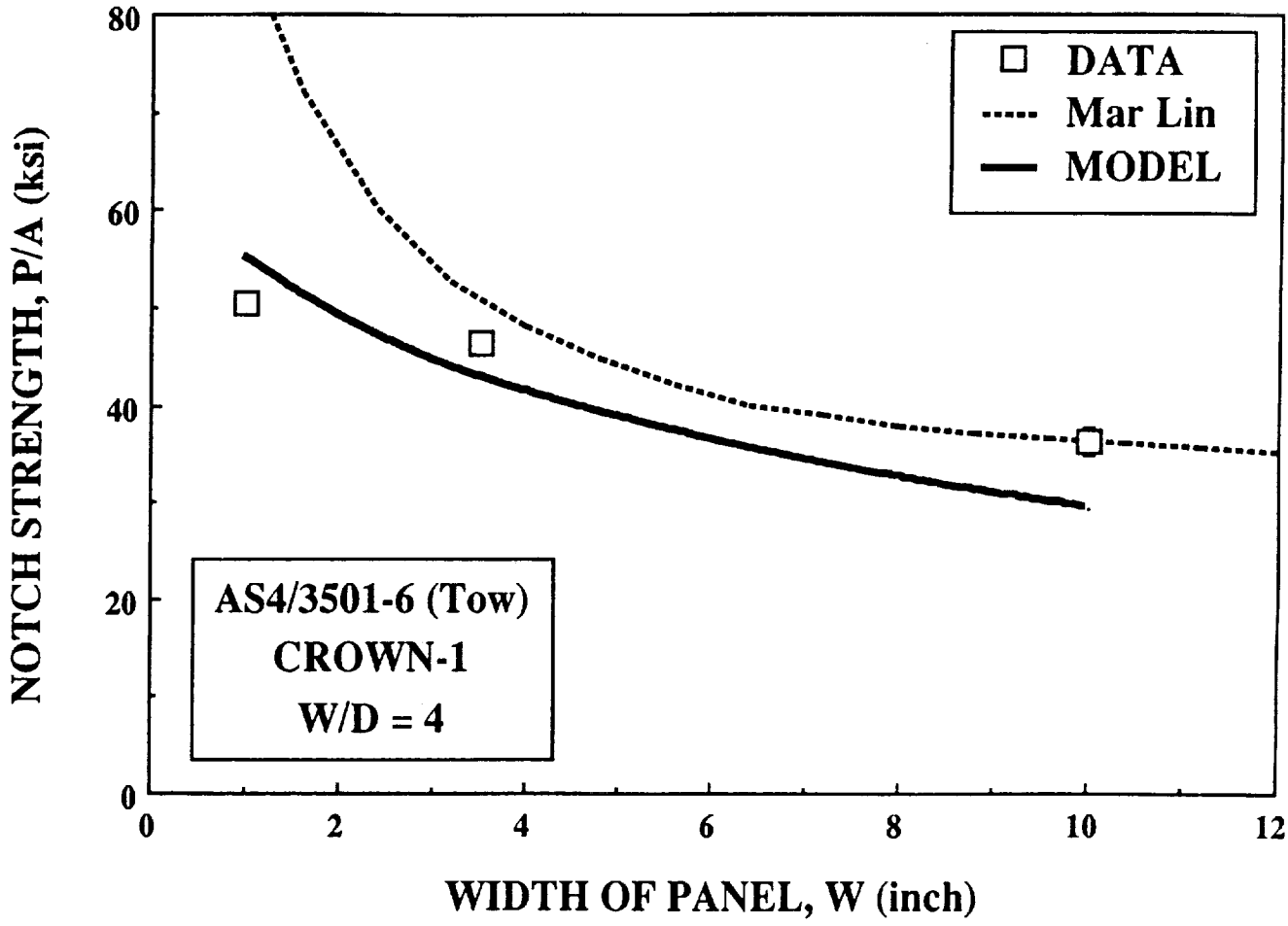
RESIDUAL STRENGTH

The residual strength distribution of notched $[0/90/0/90]_s$ composites as a function of laminate width. Comparison between the prediction and the test data.



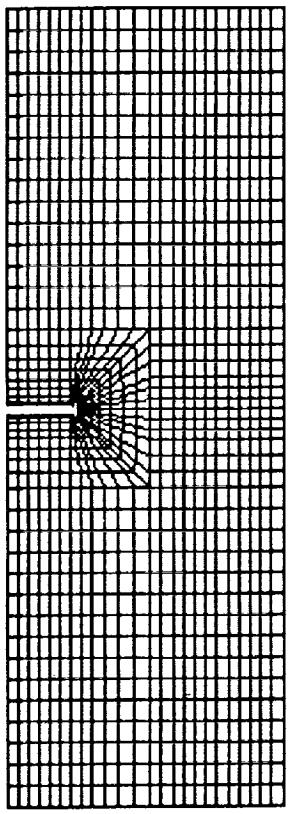
RESIDUAL STRENGTH

The residual strength distribution of notched [Crown-1] tow-composites as a function of laminate width. Comparison between the predictions based on the present model and the Mar-Lin model and the test data.



PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



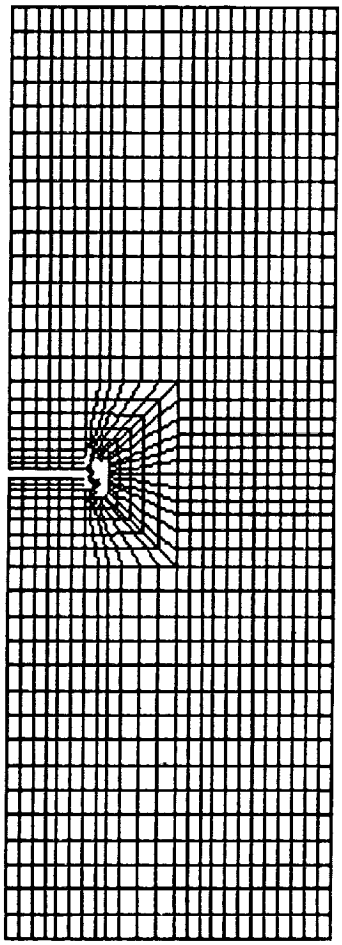
MATERIAL: AS4/3501-6 (TOW)
LAYUP: CROWN-1
D= 0.872(in)
W/D= 4.0

LOAD = 3612(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



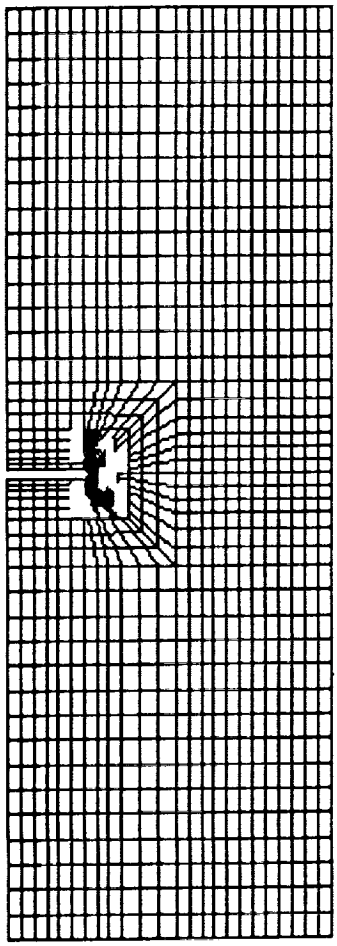
MATERIAL: AS4/3501-6 (TOW)
LAYUP: CROWN-1
D= 0.872(in)
W/D= 4.0

LOAD = 7771(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



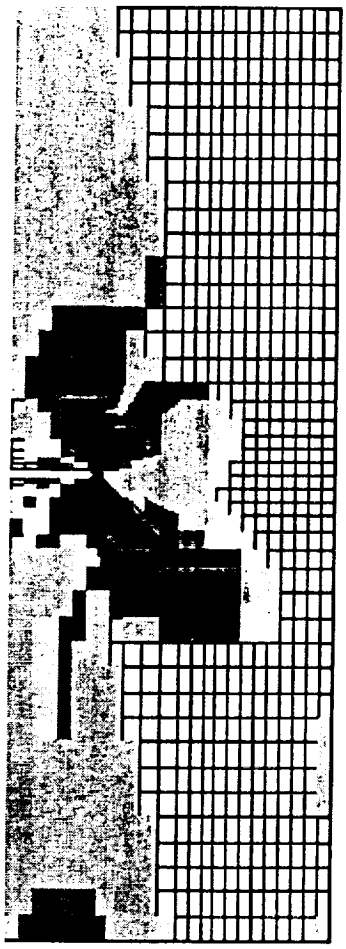
MATERIAL: AS4/3501-6 (TOW)
LAYUP: CROWN-1
D= 0.872(in)
W/D= 4.0

LOAD = 10202(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

PROGRESSIVE FAILURE PREDICTION

Numerical simulation of damaged extension of notched laminated composites as a function of applied load under uniaxial tension.



MATERIAL: AS4/3501-6 (TOW)
LAYUP: CROWN-1
D= 0.872(in)
W/D= 4.0

LOAD = 8612(lbs)

DAMAGE MODE
- Fiber Breakage
- Fiber-Matrix Shear-Out
- Matrix Cracking

I. IMPLEMENTATION

1. IMPLEMENTATION OF THE CURRENT MODEL TO EXISTING FEM CODES

II. DAMAGE MODELLING

1. CRACK GROWTH MODEL
2. DELAMINATION INITIATION AND GROWTH MODEL
3. FATIGUE MODEL

III. COMPUTATIONAL MECHANICS

1. MESH SENSITIVITY
2. DAMAGE SIMULATION
3. GLOBAL-LOCAL FEM
4. PARALLEL PROCESSING